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Transport in multi-chain models of interacting fermions

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Abstract. We study transport in a class of exactly solvable multi-chain models of interacting fermions using bosonization. We introduce magnetic and non-magnetic impurities at a site, through either weak δ-function potentials or weak links. Using a renormalization group analysis, we show that the strength of the non-magnetic impurity is not affected by the interaction, and hence, conductance through the impurity can be exactly computed. Magnetic impurities, on the other hand, in a single- or two-chain model always cut the wire at the impurity site. However, when the number of chains is greater than two, we find a critical interaction strength beyond which there exists a metal–insulator transition tuned by the strength of the magnetic impurity.

1. Introduction

Transport in strongly correlated electron systems is an emerging field, which has suddenly shot into prominence in the last few years. This is mainly due to advances in semiconductor technology, which have enabled the fabrication of one-dimensional quantum wires in the single-channel limit [1]. Also, the recent discovery of carbon nanotubes [2] and multi-probe experiments on them have spurred both theoretical and experimental activity in this area.

One-dimensional interacting electrons are known to exhibit Luttinger liquid (LL) behaviour, rather than Fermi liquid (FL) behaviour, characterized by spin–charge separation and the absence of Landau quasi-particles [3]. In the presence of impurities, transport responses are marked by interaction-dependent power laws. The dramatic effect of interactions on transport in a LL was shown in a paper by Kane and Fisher [4], who studied transport in LLs, through both single and double barrier structures. They showed through renormalization group arguments that spinless electrons with repulsive interactions, incident upon a single barrier, are completely reflected at zero temperature and for vanishing bias, whereas for attractive interactions there is perfect transmission. This analysis has been further modified by several groups [5] who included other corrections, but essentially verified their analysis. It has also been verified by an exact solution at arbitrary coupling constant using the thermodynamic Bethe ansatz [6].

Theoretically, the behaviour of coupled chains [7, 8] of LLs has also attracted a great deal of interest in the past few years, because they are expected to be relevant in interpreting the results in quasi-one-dimensional conductors. However, the issue of transport through isolated impurities in multi-chain LLs has not yet been addressed systematically. One reason has been that, at least for models such as coupled Hubbard chains, there are several possible phases, even in the small-U limit, and it is difficult even to determine the phase of the system in the large-U limit. So it is premature to try and study transport in these models. Some attempts

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have been made to couple LLs at isolated points and study transport through them, such as in the studies through crossed LLs [9] and through double and multiply crossed LLs [10]. These studies have only dealt with two coupled chains. Transport through more chains is harder to compute because essentially the number of channels for both forward and backward scattering get multiplied by the number of chains.

All of these analyses are within the bosonized Luttinger model approach [11], with strong or weak barriers, whose effects are then extrapolated using renormalization group arguments. Since the difference between transport for the FL and the LL is so dramatic, even for a single chain, it is worthwhile seeing whether such results are reproduced in other models of interacting fermions. It could also happen that it may be easier to tackle the problem of transport through isolated impurities in a many-chain system in some of these other models. Also, since any external voltage applied to the wires explicitly breaks time reversal (TR) invariance, it may be of interest to study transport in models where TR symmetry is violated. With a view to addressing the above issues, in this paper we study transport in a class of exactly solvable models of interacting fermions in one dimension, first proposed by Schulz and Shastry [12]. These models have been extended to multi-chain models [13, 14] and, in fact, the study here is one of the first studies of transport through multi-chain models of interacting fermions.

In these models, the particles essentially interact through a gauge potential, which, however, can be removed by a pseudo-unitary transformation, and the problem can be exactly solved, but it still exhibits non-trivial LL behaviour. The idea of introducing interactions through a gauge potential is not new or exotic in one dimension. In Bethe ansatz solvable problems, interactions redefine the momenta of the excitation spectrum in a non-trivial way, which is precisely what a gauge potential does. In the models that we study here, the phase that one kind of particle picks up while moving in the background of the other kind of particle is rather simple and can be parametrized in terms of a single constant. Also, these models cannot be fitted into the general framework of LLs studied by Kane and Fisher [4], because of the absence of TR symmetry and because these models do not allow a neat separation of the 'spin' and 'charge' degrees of freedom. Thus, it is of interest to see whether the general features of the transport analysis of Kane and Fisher hold in these models, particularly since finite external potentials do break TR symmetry. Moreover, since these models have some simplifying features, it is easier to study transport in the multi-chain version of these models than in the multi-chain version of the usual LL models.

These exactly solvable 'gauge' models are described in terms of two species of fermions, with pseudospin index $\sigma = \pm$ and with 'gauge' interactions described by a Hamiltonian given by

$$H = \sum_{I=1}^{N} \sum_{\sigma i} a_I (\Pi_{\sigma i})^{2I}. \tag{1}$$

Here, $\Pi_{\sigma i} = p_{\sigma i} + \sigma A_\sigma(x_{\sigma i})$ is the 'covariant momentum' introduced in [12] and N is a 'band index' introduced in [13, 14], which generalized these models by allowing for higher powers of the covariant momenta in the Hamiltonian. Particles interact with each other via the gauge potential, given for the particle at the position x by $A_\sigma(x) = \sum_j V(x - x_{-\sigma j})$, i.e. the potential for a particle with positive pseudospin is due to the presence of particles with negative pseudospin and vice versa. The potential is chosen to be an even function and vanishes at infinity. The Hamiltonian explicitly breaks TR invariance, although it is invariant under a combined operation of TR and reversal of pseudo-spin index.

Since the interactions in this model are essentially introduced through a self-generated gauge potential, we choose a ring geometry to study the problem of transport. Particles of one species (or pseudospin orientation) give rise to an effective Aharanov–Bohm flux acting on

the particles of the other species (other pseudospin orientation). However, as we shall discuss at the end of this paper, the geometry is not important and the same results can be obtained in an open wire. So, explicitly, we consider a ring of length L and impose the periodic boundary conditions (PBCs) $\psi(x) = \psi(x + L)$, and instead of measuring the current in the limit of vanishing external voltage, we measure the persistent current in the model in the limit of vanishing external flux through the ring.

In section 2, we confine ourselves to the original single-chain version of the model. We first define the current and compute the conductance of a 'clean' wire. We then introduce non-magnetic and magnetic impurities through both weak barriers and weak links and study the renormalization group flows of the appropriate parameters. We show that non-magnetic impurities lead to marginal operators and hence transport through them can be explicitly computed. However, any magnetic impurity 'cuts' the wire, i.e. the conductance is zero as the temperature $T \to 0$ and as the driving forces, V (or Φ) $\to 0$. In section 3, we study transport in the multi-band version of the model. We discuss the approximations under which the N-band version of the model is relevant to an N-chain system and show, in particular, that only for $N \leqslant 3$ we expect our model to be applicable to realistic coupled N-chain LLs. Then, once again, we introduce magnetic and non-magnetic impurities. But here, we show that, although non-magnetic impurities continue to be marginal, magnetic impurities can lead to a non-trivial fixed point. For sufficiently strong interactions, weak barriers are conducting, but strong barriers are insulating.

2. Transport in a Schulz–Shastry chain

In this section, we shall confine ourselves to the original model of Schulz and Shastry [12] and set $N = 1$ in equation (1). By performing a pseudo-unitary transformation on the model:

$$e^{iS(\{x_{+i}\},\{x_{-j}\})} p_{\sigma i} e^{-iS(\{x_{+i}\},\{x_{-j}\})} = p_{\sigma i} - \partial_{x_{\sigma i}} S(\{x_{+i}\},\{x_j\}) \tag{2}$$

the interaction term in equation (1) is eliminated and the Hamiltonian is transformed to the free one given by $H = (1/2)\sum_{\sigma i}(p_{\sigma i})^2$. However, the boundary conditions on the wavefunctions are now different and the quantization conditions for the momenta are now replaced by

$$Lk_{\pm,i} \mp N_{\mp}\delta = 2\pi n_{\pm,i}. \tag{3}$$

Here δ is a phase shift that can be computed in terms of the unitary transformation function S, which in turn is related to the original interaction, since S is chosen to cancel the interaction [12]. $n_{\pm,i}$ are quantum numbers analogous to those used in the non-interacting case. Note that $\tilde{\delta}$, which is defined as the fractional part of $\frac{\delta}{2\pi}N_{\pm}$ ($\tilde{\delta}$ takes values from 0 to 1), leads to interactions; the integer part of the ostensible interaction term $\frac{\delta}{2\pi}N_{\pm}$ merely gives integer changes in the quantum numbers $n_{\pm,i}$, which anyway take values over all integers.

We first obtain the ground state energy E_0 in a sector with $N_{\pm}$ particles by choosing $n_{\pm,i} = n^0_{\pm,i} \pm [[\frac{\delta}{2\pi}N_{\mp}]]_{\text{int}}$, where $n^0_{\pm,i}$ are the quantum numbers in the absence of any interaction. Excitations about the ground state are obtained by constructing the low-energy Hamiltonian when particles are added to the left and right Fermi points. Let us assume that the ground state has $2n_0 + 1$ particles of each kind and that $\frac{\delta}{2\pi}(2n_0 + 1)$ is an integer. The addition of $n_{\pm R}(n_{\pm L})$ particles at the right(left) Fermi points will cause a second-order change in the ground state energy given by

$$E^{(2)} = \frac{n\pi^2}{4L}\{(1+\bar{\delta}^2)(N_+^2 + N_-^2) + (J_+^2 + J_-^2) + 2\bar{\delta}(J_+N_- - J_-N_+)\} \tag{4}$$

where the initial particle density is denoted as $n = \frac{2(2n_0+1)}{L}$, $\bar{\delta} = \frac{\delta}{\pi}$, and $N_{\pm} = n_{\pm R} + n_{\pm L}$ and $J_{\pm} = n_{\pm R} - n_{\pm L}$ are the charge and current quantum numbers, respectively. Note that the

'electric charges' of the + and − particles are opposite in equation (1), since their coupling to the $U(1)$ field has opposite signs. Hence, we can define the total 'electric charge' current due to both types of particles to be $J_+ - J_-$, which is the same as the 'spin' current, and the particle number current to be $J_+ + J_-$. Similarly, we can define $N_+ - N_-$ to be the total 'electric' or 'spin' charge and $N_+ + N_-$ to be the total particle number charge.

Let us introduce boson fields $\phi_\pm$ and their conjugate momenta $\Pi_\pm = \partial_\tau \phi_\pm = \partial_x \theta_\pm$, following the notation of Shankar [15]. These are related to the (generic) charge and current densities as

$$N_\pm = \frac{L}{\sqrt{\pi}} \partial_x \phi_\pm \qquad \text{and} \qquad J_\pm = -\frac{L}{\sqrt{\pi}} \partial_x \theta_\pm. \tag{5}$$

So, the effective low-energy Hamiltonian including quantum fluctuations is now given by

$$H = \frac{n\pi}{4} \int \mathrm{d}x \, \{(1+\bar{\delta}^2)[(\partial_x \phi_+)^2 + (\partial_x \phi_-^2)] + [(\partial_x \theta_+)^2 + (\partial_x \theta_-)^2] + 2\bar{\delta}(\partial_x \theta_- \partial_x \phi_+ - \partial_x \theta_+ \partial_x \phi_-)\}. \tag{6}$$

In terms of the spin or 'electric charge' (s) and particle number (p) fields defined by $\theta_{\mathrm{s/p}} = \frac{\theta_+ - \theta_-}{\sqrt{2}} / \frac{\theta_+ + \theta_-}{\sqrt{2}}$ and $\phi_{\mathrm{s/p}} = \frac{\phi_+ - \phi_-}{\sqrt{2}} / \frac{\phi_+ - \phi_-}{\sqrt{2}}$, the Hamiltonian can be rewritten as

$$H = \frac{n\pi}{4} \int \mathrm{d}x \, \{(1+\bar{\delta}^2)[(\partial_x \phi_\mathrm{p})^2 + (\partial_x \phi_\mathrm{s}^2)] + [(\partial_x \theta_\mathrm{p})^2 + (\partial_x \theta_\mathrm{s})^2] + 2\bar{\delta}(\partial_x \theta_\mathrm{p} \partial_x \phi_\mathrm{s} - \partial_x \theta_\mathrm{s} \partial_x \phi_\mathrm{p})\}. \tag{7}$$

Note that, unlike the usual models of LLs, here the Hamiltonian is not separable in terms of the spin and particle number variables—the $\bar{\delta}$ term couples them, just as it couples the $\pm$ fields. This is yet another motivation to study transport in this model. Until now, the Kane–Fisher results [4] have only been obtained in models where spin and particle number are explicitly separable. Note also that we may work either with spin and particle number fields ($\alpha = \mathrm{s/p}$ in equation (9)) or with + and − fields ($\alpha = \pm$ in equation (9)).

The Hamiltonian can be brought to a form where it looks almost non-interacting by defining new fields:

$$\tilde{\theta}_\pm = \theta_\pm \mp \bar{\delta} \phi_\mp \qquad \text{and} \qquad \tilde{\phi}_\pm = \phi_\pm$$

or

$$\tilde{\theta}_{\mathrm{p/s}} = \theta_{\mathrm{p/s}} \pm \bar{\delta} \phi_{\mathrm{s/p}} \qquad \text{and} \qquad \tilde{\phi}_{\mathrm{p/s}} = \phi_{\mathrm{p/s}}. \tag{8}$$

The new fields $\tilde{\theta}_\pm$ and $\tilde{\phi}_\pm$ ($\tilde{\theta}_{\mathrm{p/s}}$ and $\tilde{\phi}_{\mathrm{p/s}}$) satisfy the same commutation relations as the non-tilde fields, $[\tilde{\theta}_\alpha(x), \tilde{\phi}_\alpha(y)] = \mathrm{i}\Theta(x-y) = [\theta_\alpha(x), \phi_\alpha(y)]$ ($\alpha = \pm$ or p/s). In terms of these fields, the Hamiltonian apparently appears to be spin-particle-number (or equivalently +/−) separable and is given by

$$H = \frac{n\pi}{4} \sum_{\alpha = \pm \text{ or } \mathrm{p/s}} \int \mathrm{d}x \, \{(\partial_x \tilde{\phi}_\alpha)^2 + (\partial_x \tilde{\theta}_\alpha)^2\}. \tag{9}$$

Either one of the fields can be treated as the coordinate and the gradient of the other field can be treated as the momentum.

2.1. Conductance for a clean wire

When there are no impurities, i.e. for a clean system, there are two conductances which can be defined—conductance for the particle number current and that for the spin–current. Since the $U(1)$ gauge field for the exactly solvable model couples to the spin–current, here if we

want to compute the conductance as a response to an externally applied $U(1)$ field, it is the spin–current conductance that we need to compute. However, here we shall compute both the conductances by defining an appropriate field even for the particle current. The currents $I_{\mathrm{p/s}}$ are related to the appropriate fields as

$$I_{\mathrm{p/s}}(x) = \int_0^L \mathrm{d}x' \int \frac{\mathrm{d}\omega}{2\pi} \mathrm{e}^{-\mathrm{i}\omega t} \sigma_{\mathrm{p/s},\omega}(x, x') E_{\mathrm{p/s},\omega}(x') \tag{10}$$

where $E_{\mathrm{p/s},\omega}(x')$ is the frequency ω component of the time Fourier transform of the appropriate electric field. The conductivity $\sigma_{\mathrm{p/s},\omega}(x, x')$, in turn, is related to the (imaginary time) current–current correlation function by the usual Kubo formula as

$$\sigma_{\mathrm{p/s},\omega}(x, x') = \frac{e^2}{\bar{\omega}} \int_0^\beta T_\tau \langle j_{\mathrm{p/s}}(x, \tau) j_{\mathrm{p/s}}(x', 0) \rangle \mathrm{e}^{-\mathrm{i}\bar{\omega}\tau} \frac{\mathrm{d}\tau}{2\pi} \tag{11}$$

where $\tau = \mathrm{i}t$, $\omega = \mathrm{i}\bar{\omega} + \epsilon$ and T_τ is the (imaginary) time-ordering operator. Using the current densities $j_{\mathrm{p/s}} = \frac{J_{\mathrm{p/s}}}{L} = \frac{1}{\sqrt{\pi}} \partial_x \theta_{\mathrm{p/s}}$, which in turn can be written in terms of the tilde fields as $j_{\mathrm{p/s}} = \frac{1}{\sqrt{\pi}} \partial_x (\tilde{\theta}_{\mathrm{p/s}} \mp \bar{\delta} \partial_x \phi_{\mathrm{s/p}})$, we find that

$$\sigma_{\mathrm{p/s},\bar{\omega}}(x, x') = \mathrm{e}^{-|\bar{\omega}||x-x'|} (1 + \bar{\delta}^2) \frac{e^2}{\pi}. \tag{12}$$

When this is used in equation (10), it yields the conductances $G_{\mathrm{p/s}} = I_{\mathrm{p/s}}/(V_{\mathrm{p/s},L} - V_{\mathrm{p/s},0})$ (where $(V_{\mathrm{p/s},L} - V_{\mathrm{p/s},0}) = \int_0^L E_{\mathrm{p/s},\omega}(x')\,\mathrm{d}x'$) as

$$G_{\mathrm{p/s}} = \frac{e^2}{h}(1 + \bar{\delta}^2) \tag{13}$$

where we have explicitly restored $\hbar$, which is usually set to one. Note that, for $\bar{\delta} = 0$, these conductances are just e^2/h, which agrees with the expected result for two free degrees of freedom. Note also that the conductances even for a clean wire do depend on the interaction as expected for a Luttinger wire. However, if the leads are FLs, as is true for the usual Luttinger wire [5], here too the interaction effect will be wiped out and we would get just e^2/h.

2.2. Conductance through non-magnetic impurities

We now study the effects of introducing a non-magnetic (or non-spin-flip) impurity in this bosonized model. Both in the weak barrier and weak link limits, we shall see that the barrier or link operator remains marginal. Hence, the impurity problem is exactly solvable and we shall obtain the results for transmission and reflection through the impurity.

2.2.1. Weak barrier limit. Let us first consider scattering from a small barrier. A potential scatterer or impurity in the model can be introduced as

$$\delta H = \int V(x)(\psi_+^\dagger(x)\psi_+(x) + \psi_-^\dagger(x)\psi_-(x))\,\mathrm{d}x \simeq \lambda(\psi_+^\dagger(0)\psi_+(0) + \psi_-^\dagger(0)\psi_-(0)) \tag{14}$$

where $V(x) \neq 0$ for a small region around the origin ($\simeq \lambda\delta(x)$) and where the first and second terms backscatter + and − particles, respectively. In momentum space, this term translates into $2k_F$ (and higher momentum transfer) scatterings between the left and right Fermi points for both the + and − particles. The lowest-order $2k_F$ scattering term is given by

$$\delta H_1 = V_{\mathrm{e}} \sum_{\sigma=\pm} \psi_{\sigma,L}^\dagger \psi_{\sigma,R} + \mathrm{h.c.} \tag{15}$$

where either a + or − particle is backscattered. Here, both charge and spin degrees of freedom are involved. However, we can also consider higher-order scatterings (e.g. $4k_F$ scattering) where both + and − particles are scattered such as

$$\delta H_2 = (V_\mathrm{p}\psi^\dagger_{+,L}\psi_{+,R}\psi^\dagger_{-,L}\psi_{-,R} + V_\mathrm{s}\psi^\dagger_{+,L}\psi_{+,R}\psi^\dagger_{-,R}\psi_{-,L}) + \mathrm{h.c.} \tag{16}$$

In the first(second) term in δH_2 both kinds of particles are incident from the same (different) direction. Thus the spin (particle number) momentum is unaffected and hence, as we shall see, the corresponding bosonized operator depends only on particle number (spin) fields. (Note that V_e, V_s and V_p are all known in terms of $V(x)$.)

We will study the effects of both the $2k_F$ and $4k_F$ scatterings using bosonization and the renormalization group. The fermions may be bosonized using

$$\psi_{R/L}(x) = \lim_{\alpha\to 0}\frac{1}{(2\pi\alpha)^{1/2}}\mathrm{e}^{\pm\mathrm{i}\sqrt{4\pi}\phi_{R/L}} = \lim_{\alpha\to 0}\frac{1}{(2\pi\alpha)^{1/2}}\mathrm{e}^{\mathrm{i}\sqrt{\pi}(\pm\phi-\theta)} \tag{17}$$

(R goes with + and L with − in both equalities) where α is a regularization parameter, which is finally set to zero. (We ignore Klein factors, because we are using the notation of Shankar [15] which does not require the use of Klein factors. For an explicit comparison of different bosonization schemes, see [16].) Thus, the quadratic and the quartic fermion operators are bosonized as

$$O_\pm = \psi^\dagger_{\pm,L}\psi_{\pm,R} = \frac{1}{2\pi\alpha}\mathrm{e}^{\mathrm{i}\sqrt{4\pi}\phi_\pm} = \frac{1}{2\pi\alpha}\mathrm{e}^{\mathrm{i}\sqrt{2\pi}(\phi_\mathrm{p}\pm\phi_\mathrm{s})} \tag{18}$$

and

$$O_\mathrm{p/s} = \psi^\dagger_{+,L}\psi_{+,R}\psi^\dagger_{-,L/R}\psi_{-,R/L} = \frac{1}{(2\pi\alpha)^2}\mathrm{e}^{\mathrm{i}\sqrt{8\pi}\phi_\mathrm{p/s}}. \tag{19}$$

Note that, as we had expected, the two-fermion operators depend both on charge and spin, whereas the four-fermion operators involve either the charge or the spin.

The renormalization group flows of these operators to leading order can be found by computing their dimensions. Since the operators depend on the ϕ fields, their dimensions are conveniently computed using ϕ as the coordinate field. For the bilinear operators, the dimensions are computed as

$$\langle O^\dagger_\pm(\tau,0)O_\pm(0,0)\rangle = \frac{1}{(2\pi\alpha)^2}\left(\frac{\alpha^2}{\alpha^2+\tau^2}\right) \tag{20}$$

whereas for the operators that are quartic in the Fermi fields, we get

$$\langle O^\dagger_\mathrm{p/s}(\tau,0)O_\mathrm{p/s}(0,0)\rangle = \frac{1}{(2\pi\alpha)^4}\left(\frac{\alpha^2}{\alpha^2+\tau^2}\right)^2. \tag{21}$$

Clearly, the correlation functions diverge as τ^{-2} for $O_\pm$ and τ^{-4} for $O_\mathrm{p/s}$ as $\tau \longrightarrow 0$. This leads to the RG equations:

$$\frac{\mathrm{d}V_\mathrm{e}}{\mathrm{d}t} = 0 \qquad \text{and} \qquad \frac{\mathrm{d}V_\mathrm{p/s}}{\mathrm{d}t} = -V_\mathrm{p/s}. \tag{22}$$

Thus, $O_\pm$ are marginal and $O_\mathrm{p/s}$ are irrelevant, just as they are in the non-interacting case. Generically, once the barrier becomes stronger, the above leading order analysis of the RG flow is no longer valid, and it is necessary to study the RG flow to higher orders in V. Since only one operator is marginal and the rest are irrelevant, one needs to ask whether the marginal operator is marginally relevant or marginally irrelevant when computed to higher orders in V_e. But here, it is easy to check that the operator $\cos\phi_\pm$ can never contribute to itself at any order in perturbation theory, because $\cos^n(\sqrt{4\pi}\phi_\pm)$ does not contain a factor $\cos(\sqrt{4\pi}\phi_\pm)$ for any

$n > 1$. Hence, the operator in equation (18) or the $2k_F$ backscattering remains marginal to all orders in V_e and all higher-order scattering processes ($4k_F$, $6k_F$, . . ., etc) are irrelevant.

One may still wonder whether the RG flow equation for the $2k_F$ backscattering can receive non-perturbative contributions. But in this model, since the '+ particles' interact only with '− particles' and vice versa, the backscattering of any one kind of particle is just not affected by the interaction. Another way of seeing this is to note that this interacting system differs from the non-interacting one by a pseudo-unitary transformation which does not affect the diagonal density operator $\psi_\sigma^\dagger \psi_\sigma$. Hence, even non-perturbative corrections to the lowest-order flow equations cannot occur.

2.2.2. Strong barrier limit. However, we can also see this explicitly if, following Kane and Fisher [4], we study the model perturbatively from the other limit, i.e. the case where there is an infinite barrier with no transmission across the barrier. (We do this here, although it is not necessary, to set the notation for when we introduce the magnetic impurity, where it is necessary.) Then, the fact that the barrier is not infinite is modelled by a hopping term across the site. We then use perturbative RG to determine whether the hopping term is relevant, leading to a 'healing' of the wire, is irrelevant, confirming the stability of the infinite barrier fixed point, or is marginal.

In first quantized language, for an infinite barrier, the wavefunction ψ_σ has to vanish at the impurity site $x = 0$. This allows only odd parity states ($\frac{1}{2}[\psi_\sigma(x) - \psi_\sigma(-x)]$). In terms of the bosonized fields, the condition of infinite barrier at the origin is imposed in the following way. We bosonize the $0 < x < L/2$ section by introducing the bosonic fields $\phi_{\alpha>}$ and $\theta_{\alpha>}$, and the $0 > x > -L/2$ section by the fields $\phi_{\alpha<}$ and $\theta_{\alpha<}$. (The ring boundary condition is satisfied by demanding continuity of charge and current at $-L/2 \equiv +L/2$.) The Hamiltonian for the infinite barrier case is then given by $H = \sum_i H_i$, with $i = >, <$ where,

$$H_i = \frac{n\pi}{4} \int \mathrm{d}x \left\{ (1+\bar{\delta}^2)[(\partial_x \phi_{+i})^2 + (\partial_x \phi_{-i}^2)] + [(\partial_x \theta_{+i})^2 + (\partial_x \theta_{-i})^2] \right.$$
$$\left. + 2\bar{\delta} \left(\partial_x \theta_{-i} \sum_j \partial_x \phi_{+j} - \partial_x \theta_{+i} \sum_j \partial_x \phi_{-j} \right) \right\} \tag{23}$$

with the boundary conditions $\phi_<(0) = \phi_>(0) = \frac{\pi}{2}$ [7]. Although we have effectively decoupled the wire by allowing for two different kinds of bosons $<$ and $>$ to the left and right of the origin, respectively, note that the $\sum_j \partial_x \phi_{\sigma j}$ terms in the Hamiltonian ensures that the '+' particles in any one section ($>$ or $<$) interacts with all '−' particles ($<$ and $>$) in the ring [13]. This would certainly not have been true if we had two disconnected wires (rings). Since the interaction energy is not extensive, the original ring can never be reproduced starting from two completely disconnected rings. The above Hamiltonian, in fact, models a single ring with no current across the impurity. (Here we wrote the Hamiltonian in $\alpha = \pm$ representation, but equivalently, we could have written it in terms of the spin and charge fields.) A redefinition of both the $<$ and $>$ fields, analogous to the field redefinitions in equation (8), brings it to the apparently non-interacting form given by

$$H = \frac{n\pi}{4} \sum_{\sigma,i} \int \mathrm{d}x \{ (\partial_x \tilde{\phi}_{\sigma i})^2 + (\partial_x \tilde{\theta}_{\sigma i})^2 \} \tag{24}$$

with $\sigma = \pm$ or p, s and $i = <, >$.

We now introduce a hopping term across $x = 0$, as $\delta H = t \sum_\sigma \bar{O}_\sigma$, where

$$\bar{O}_\sigma = \psi_{\sigma>}^\dagger(0) \psi_{\sigma<}(0) + \text{h.c.}$$

$$= \frac{1}{(2\pi\alpha)} e^{i\sqrt{\pi}(\theta_{\sigma>}(0)-\theta_{\sigma<}(0))} + \text{h.c.}$$
$$= \frac{1}{(2\pi\alpha)} e^{i\sqrt{\pi}(\tilde{\theta}_{\sigma>}(0)-\tilde{\theta}_{\sigma<}(0))} + \text{h.c.} \tag{25}$$

and t is an overlap matrix element. Note that, since the hopping operator depends only on the $\theta_\sigma(0)$ (since $\phi_\sigma(0) = 0$), we may compute its dimension using the Hamiltonian in equation (24) by treating the $\tilde{\theta}$ fields as the coordinate fields. The dimensions of the hopping operators are easily computed as

$$\langle \bar{O}_\sigma^\dagger(\tau, 0)\bar{O}_\sigma(0, 0)\rangle = \frac{1}{(2\pi\alpha)^2}\left(\frac{\alpha^2}{\alpha^2+\tau^2}\right) \tag{26}$$

which says that the operator is marginal, as expected, and hence the single-particle backscattering process ($2k_F$) does not depend on the interaction. As for the non-interacting case, it depends on the strength of the scattering potential and the energy of the incident particles. In this limit too, one can show explicitly that the higher-order processes are irrelevant.

Hence, the result that we obtain for transmission when the wire has a non-magnetic impurity at the origin introduced either through a weak δ-function barrier or in the opposite limit, through weak hopping, is that the strength of the non-magnetic impurity V is unaffected by the interaction.

2.2.3. Exact results for arbitrary barrier strength. Since the system behaves like a non-interacting electron system in terms of the tilde variables, and since the impurity term is marginal, the model can be exactly solved for both the conductances, even in the presence of the impurity. We get $G_{\text{p/s,imp}} = G_{\text{p/s,clean}}T$, where T is the transmission probability of a electron with energy E_F. For a δ-function potential of strength λ at the origin, we have

$$G_{\text{p/s,imp}} = \frac{2e^2}{h}(1+\delta^2)\frac{E_F}{(E_F+\lambda^2)}. \tag{27}$$

Thus, depending on the values of E_F and λ, a part of the incident electron gets transmitted and a part gets reflected. Neither perfect transmission nor perfect reflection is observed.

2.2.4. An aside. As an aside, note that the dimension of the same hopping operator can also be computed in a system of two completely disconnected rings. The Hamiltonian for such a system is the same as equation (24), except that the $\sum_j \partial_x\phi_{\sigma j}$ is replaced by $\partial_x\phi_{\sigma i}$. In this model, the dimensions are given by

$$\langle \bar{O}_\sigma^\dagger(\tau, 0)\bar{O}_\sigma(0, 0)\rangle = \frac{1}{(2\pi\alpha)^2}\left(\frac{\alpha^2}{\alpha^2+\tau^2}\right)^{1+\bar{\delta}^2} \tag{28}$$

which implies that the operator is irrelevant. So the model with two disconnected rings does *not* get healed; rather, it flows to the insulating fixed point where the two rings are disconnected.

2.3. Conductance through magnetic impurities

What about impurities which can flip the spin of the electron at the impurity site? Such magnetic impurities can be introduced through spin-dependent potentials.

2.3.1. Weak barrier limit. For a weak barrier, we may introduce

$$\delta H = \tilde{V}(\psi_+^\dagger(0)\psi_-(0) + \text{h.c.}) \tag{29}$$

At the lowest order, this will generate two different $2k_F$ backscattering processes, $\psi_{+,L}^\dagger \psi_{-,R}$ and $\psi_{+,R}^\dagger \psi_{-,L}$, both of which can be written in the bosonized form as

$$\tilde{O}_\pm = \psi_{\pm,L}^\dagger \psi_{\mp,R} = \mathrm{e}^{\mathrm{i}\sqrt{\pi}(\phi_\pm + \phi_\mp + \theta_\pm - \theta_\mp)} \tag{30}$$

where $\tilde{O}_\pm$ is just the notation for the operators $\psi_{+,L}^\dagger \psi_{-,R}$ and $\psi_{-,L}^\dagger \psi_{+,R}$, respectively. Since for computing dimensions, it is more convenient to use the apparently non-interacting form of the Hamiltonian given in equation (9), these operators may be rewritten in terms of the tilde fields as

$$\tilde{O}_\pm = \mathrm{e}^{\mathrm{i}\sqrt{\pi}[(1\pm\bar{\delta})\tilde{\phi}_\pm + (1\pm\bar{\delta})\tilde{\phi}_\mp + \tilde{\theta}_\pm - \tilde{\theta}_\mp]} \tag{31}$$

using equation (8). The lowest-order contribution to the RG equation for V is now simply obtained by computing the dimension of the operator and the correlation function is given by

$$\langle \tilde{O}_\pm^\dagger(\tau, 0)\tilde{O}_\pm(0, 0)\rangle = \frac{1}{(2\pi\alpha)^2}\left(\frac{\alpha^2}{\alpha^2 + \tau^2}\right)^{\frac{1}{2}[1+(1\pm\bar{\delta})^2]} \tag{32}$$

which diverges as $\tau^{-2-\bar{\delta}^2 \mp 2\bar{\delta}}$ as $\tau \longrightarrow 0$. So the operator $\tilde{O}_+ = \psi_{+,L}^\dagger \psi_{-,R}$ is relevant for $\bar{\delta} < 0$, whereas the operator $\tilde{O}_- = \psi_{-,L}^\dagger \psi_{+,R}$ is relevant for $\bar{\delta} > 0$. Thus one of the two backscattering spin-flip operators is always relevant for either sign of effective interaction strength. Hence, the magnetic impurity cuts the wire at the site of the impurity. Since the lowest-order $2k_F$ backscattering is relevant, we need not look for higher-order processes. An explicit check, however, shows that all such operators are irrelevant.

2.3.2. Strong barrier limit. Once the impurity strength grows under renormalization, the weak barrier analysis is no longer valid. As was done in the case with non-magnetic impurities, we need to study the strong barrier limit, where we start from the insulating limit and then study weak hopping. We take the Hamiltonian of equation (23) in the θ representation (or equation (24) in the $\tilde{\theta}$ representation) which is effectively cut at the site of impurity as the unperturbed Hamiltonian. The hopping of electrons across the the weak link, accompanied by a spin flip, is modelled by

$$\delta H = \tilde{t}(\psi_{+>}^\dagger(0)\psi_{-<}(0) + \text{h.c.})$$

which can be bosonized as follows:

$$\delta H = \frac{\tilde{t}}{2\pi\alpha}\mathrm{e}^{\mathrm{i}\sqrt{\pi}(\theta_{+>} - \theta_{-<})} + \text{h.c.} = \frac{2\tilde{t}}{2\pi\alpha}\cos\left[\sqrt{\pi}\left(\tilde{\theta}_{+>} - \tilde{\theta}_{-<} - \bar{\delta}\sum_{\sigma,i}\tilde{\phi}_{\sigma,i}\right)\right] \tag{33}$$

where the apparently non-interacting tilde fields have been used in the second equality. The scaling dimension can easily be read off as $1 + 2\bar{\delta}^2$. So hopping remains irrelevant for all values (both + and −) of the interaction parameter. Here again, we can explicitly check that all higher-order processes continue to be more irrelevant. The wire which is cut at the weak barrier limit does not get 'healed' for any strength of the barrier.

2.4. Results and discussion

This interacting electron system thus flows towards an insulating fixed point in the presence of magnetic impurity. Before going to the multi-band system, let us contrast these results with the results obtained in a purely TR invariant model of a LL, e.g. the model studied by

Kane and Fisher [4]. They found that, for spinless fermions, repulsive interactions imply that any barrier cuts the wire and attractive interactions imply that any barrier is rendered invisible in the renormalization group sense. However, once both spins and charges are introduced with arbitrary values of g_ρ and g_σ denoting the strengths of the interactions of the charge and spin sectors, various phases do exist. Besides the purely conducting and the purely insulating phases, mixed phases where there exists a non-trivial fixed point depending on barrier height separating the two phases also exist. Kane and Fisher also showed how to access the non-trivial fixed points by studying the renormalization group equations to second order in the impurity strengths around the weak and the strong barriers.

As mentioned earlier, this model cannot be fitted into the general framework of the Kane–Fisher models of LLs, because it explicitly violates TR symmetry. However, at a qualitative level, the results are similar to the Kane–Fisher results [4] for repulsive interactions. At least for spin-flip impurities, one finds that any barrier, however small, cuts the wire. For non-magnetic or spin-conserving impurities, however, we found that this model is similar to a non-interacting model, and allows both transmission and reflection. Note that we are not assuming phase coherence through the whole ring as exists for mesoscopic systems. Hence, our results are directly relevant also to the case of open wires, because the boundary condition that $\psi(L/2) = \psi(-L/2)$, which needed to be imposed for the ring, plays no role for transmission through the impurity. But such coherent systems have also been studied for non-interacting fermions with novel results [17]; extension of those ideas to interacting rings, however, is non-trivial [19].

3. Transport in multi-band models

Now let us study transport in a multi-band LL. The Hamiltonian (equation (1) with $N \neq 1$) is a power series of the covariant momenta $\Pi_{\sigma,i}$ and can easily be diagonalized by a similar pseudo-unitary transformation. This leads to at most $2N$ channels of excitations around the N left and the N right Fermi points [13]. As before, we add $n_{L\sigma I}$ and $n_{R\sigma I}$ particles for the left and right movers of the Ith branch at the appropriate Fermi points and compute the second-order change in the ground state energy. Since the details are available [13], here we directly write down the bosonic Hamiltonian given by

$$H = \sum_{\sigma I} \int \mathrm{d}x\, \rho_I \{(\partial_x \tilde{\phi}_{\sigma I})^2 + (\partial_x \tilde{\theta}_{\sigma I})^2\}$$
$$\text{with} \quad \tilde{\phi}_{\sigma I} = \phi_{\sigma I} \qquad \text{and} \qquad \tilde{\theta}_{\sigma I} = \theta_{\sigma I} - \sigma \bar{\delta} \sum_I \phi_{-\sigma I} \tag{34}$$

in terms of the tilde fields. ρ_I is the effective density of particles in each band. The redefinition of $\tilde{\theta}_{\sigma I}$ in the above equation explicitly shows that one kind of particles in a particular channel interacts with all the other kind of particles in *all* the bands.

3.1. Conductance of a clean multi-channel system

We study transport in this clean system in the same way as was done for the single band model. The total particle and spin currents are defined to be $J_\mathrm{p} = \sum_i (\partial_x \theta_{+i} + \partial_x \theta_{-i}) = \sum_i [\tilde{\partial}_x \theta_{+i} + \tilde{\partial}_x \theta_{-i} + N\bar{\delta}(\phi_{-i} - \phi_{+i})]$ and $J_\mathrm{s} = \sum_i (\partial_x \theta_{+i} - \partial_x \theta_{-i}) = \sum_i [\tilde{\partial}_x \theta_{+i} - \tilde{\partial}_x \theta_{-i} + N\bar{\delta}(\phi_{-i} + \phi_{+i})]$. Hence the corresponding conductances, which directly depend on the current–current correlation functions, can easily be found using the Kubo formulae. We find that

$$G^N_{\mathrm{p/s}} = N \frac{2e^2}{h} (1 + N^2 \bar{\delta}^2). \tag{35}$$

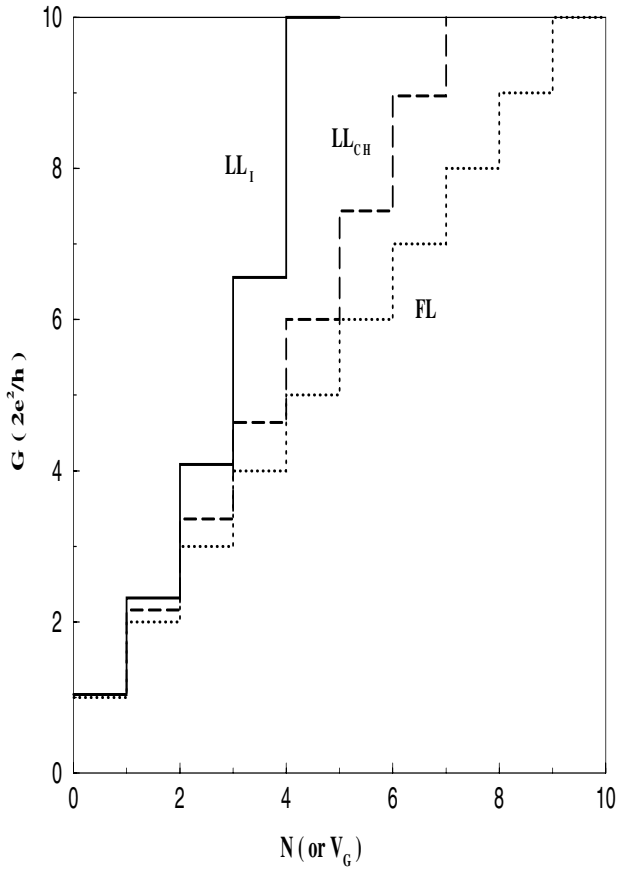

Figure 1. Conductance of a clean system is plotted versus N or equivalently the gate voltage $V_{\rm G}$ for (i) (dotted line) non-interacting system(or FLs, FL), (ii) (broken line) for LLs coupled by hopping $\rm LL_{CH}$, i.e. no interchannel interaction and (iii) (full line) for interacting LLs, $\rm LL_I$.

The point to note is that the conductances are not just the sum of N single-channel conductances, as would have been the case if the different channels did not see each other. In that case, the '+' particles of one channel would not see the '−' particle of the other channels, and $\tilde{\Pi}_{\sigma i}$ would have been defined as $\tilde{\Pi}_{\sigma i} = \Pi_{\sigma i} - \sigma\,\partial_x\phi_{\sigma i}$. In that case, the conductances would be $G^N_{\rm p/s} = N\frac{2e^2}{h}(1+\bar{\delta}^2)$. The differences can be clearly seen in figure (1), where we have plotted the following N-channel conductances: (1) N non-interchannel and non-intrachannel interacting (i.e. FL) systems, (2) N non-interchannel interacting but intrachannel interacting ($\rm LL_{CH}$) systems and (3) N interchannel as well as intrachannel interacting ($\rm LL_I$) systems.

3.2. Conductance through impurities

Now to introduce a single impurity in this multi-channel model, we first note that the fermion field is given by

$$\psi_\sigma(x) \sim \sum_I (\mathrm{e}^{\mathrm{i}k_{FI}x}\psi_{R\sigma I}(x) + \mathrm{e}^{-\mathrm{i}k_{FI}x}\psi_{L\sigma I}(x)) \tag{36}$$

where I is the channel index and the sum is over all the channels, and k_{FI} are the N Fermi points. (For details, see [13].) By substituting this in either equation (14), which is the impurity

potential for a non-magnetic impurity, or in equation (29), which is the impurity potential for a magnetic impurity, we find that backscattering of electrons occurs from one band to all the other bands (including to itself). We study the RG flow equations of all these operators. The operators corresponding to non-magnetic impurities are given by $V^{\pm}_{IJ}\psi^{\dagger}_{R\pm I}(0)\psi_{L\pm J}(0)$ and those corresponding to magnetic impurities are given by $\tilde{V}^{\pm}_{IJ}\psi^{\dagger}_{R\pm I}(0)\psi_{L\mp J}(0)$. The RG equations for the $V^{\pm}_{IJ}$ and the $\tilde{V}^{\pm}_{IJ}$ are given by

$$\frac{\mathrm{d}V^{\pm}_{IJ}}{\mathrm{d}t} = 0 \qquad \text{and} \qquad \frac{\mathrm{d}\tilde{V}^{\pm}_{IJ}}{\mathrm{d}t} = \left(\pm\bar{\delta} - \frac{N}{2}\bar{\delta}^2\right)\tilde{V}^{\pm}_{IJ}. \tag{37}$$

Like in the one-band model, the strength of the non-magnetic impurities are unaffected by the interaction. They are marginal operators and, in their presence, the system behaves like a non-interacting system. Both the spin and particle conductances can be computed exactly and for a δ-function potential with strength λ we find that

$$G^{N}_{\mathrm{p/s,imp}} = N\frac{2e^2}{h}(1 + N^2\bar{\delta}^2)\frac{E_F}{E_F + \lambda^2}. \tag{38}$$

The magnetic impurity strengths $\tilde{V}^{\pm}_{IJ}$, on the other hand, have a non-trivial flow. If any one of them flows to infinity, the wire gets cut at the impurity site. It is easy to check from the RG equations that all the $\tilde{V}^{\pm}_{IJ}$ grow when $|\bar{\delta}| < \frac{2}{N}$ and the wire gets cut, just as it does for the single-band case. But for $|\bar{\delta}| > \frac{2}{N}$, all the $\tilde{V}^{\pm}_{IJ}$ flow to zero. This is unlike the single-band case, where any magnetic impurity was bound to cut the wire. Here, we find that, for $N > 2$, we can have perfect transmission for $|\bar{\delta}| > \frac{2}{N}$. This critical $\bar{\delta}$ happens to be the same as the critical $\bar{\delta}_c$ beyond which we found earlier [13] that the LL state is robust, i.e. undisturbed by any kind of perturbation. Thus, it appears that for $|\bar{\delta}| > \frac{2}{N}$ this interacting system does not respond to weak perturbations of any kind, either global or boundary perturbations (i.e. perturbations at a single point).

To finish the RG analysis, we also need to check the results from the strong barrier (or weak link) side. As before, to model the 'cut' wire we first take two different fields $\psi_{<}(x)$ and $\psi_{>}(x)$ in the regions $-\frac{L}{2} < x < 0$ and $0 \leqslant x < \frac{L}{2}$, respectively, with the boundary condition $\psi_{<}(-\frac{L}{2}) = \psi_{>}(\frac{L}{2})$ and impose the constraint that both the fields have to vanish at the impurity site. The conditions at the origin, $\psi_{>\pm I}(0) = 0 = \psi_{<\pm I}(0)$, translate to $\phi_{>\pm I}(0) = \frac{\pi}{2} = \phi_{<\pm I}(0)$ for the bosonic fields. We then introduce hopping from one region to the other with a small hopping amplitude $\tilde{t}^{\pm}_{IJ}\psi^{\dagger}_{>R\pm I}(0)\psi_{<R\mp J}(0)$ and study the flow of $\tilde{t}$. To leading order, we find that

$$\frac{\mathrm{d}\tilde{t}^{\pm}_{IJ}}{\mathrm{d}t} = -2N\bar{\delta}^2\tilde{t}^{\pm}_{IJ}. \tag{39}$$

Surprisingly, independent of the strength of the interaction, $t^{\pm}_{IJ}$ is always irrelevant. This implies that a strong magnetic impurity *always* cuts the wire. This is different from what we might have naively expected from the weak coupling analysis, which told us that, for $|\bar{\delta}| > \frac{2}{N}$, the wire is expected to be healed! So for this range of $|\bar{\delta}|$, at some intermediate value of the barrier strength, there must be a non-trivial fixed point, which changes the behaviour from conducting to insulating.

3.3. Results

Let us summarize the results that we have found in this section. Non-magnetic impurities in this class of models are always marginal and hence allow both reflection and transmission as a function of the barrier strength and independent of the interaction strength. But magnetic

impurities lead to a far richer phase diagram. For a single-channel or a two-channel model, any magnetic impurity always cuts the wire, independent of the strength of the impurity and independent of the strength of the interactions. However, for large N, beyond a critical interaction strength $\bar{\delta}_c = \frac{2}{N}$ the system is a Luttinger metal (wire is healed) for a weak barrier and a perfect insulator (wire is cut) for a strong one. So for this interaction range ($\frac{2}{N} < \bar{\delta} < 1$), there must exist a non-trivial fixed point separating the metallic phase from the insulating one, which should depend on the strength of the magnetic impurity potential.

4. Relevance to N-chain models

What is the relevance of the N-band models that we have studied here to the coupled N-chain or ladder models which are of experimental relevance and have been studied for several years? For instance, the N-chain Hubbard model has been extensively discussed by Lin *et al* [8] in the weak coupling limit. They introduced single-particle hoppings between neighbouring Hubbard chains and studied the band transitions in a weakly interacting system. They obtained the phase boundaries of various $C_x S_y$ phases (x gapless charge and y gapless spin modes) by varying the filling factor n and the interchain hopping strength $t_\perp$ and have plotted the phase diagram in the plane of n versus $\frac{t_\perp}{t}$. The *fully* gapless LL phase, $C_N S_N$ for an N-chain system, appears in a narrow zone in this phase space, up to $N = 2$ with PBC and up to $N = 3$ with open boundary condition (OBC). Thus, for $N > 2$ for PBC and for 3 for OBC, there exists no metallic phase (or phase where all the spin and charge modes are gapless). The non-existence of a metallic state inhibits the study of transport in large-N Hubbard chains. However, in the class of models studied here, the LL phase persists for large N. In fact, it is more robust for $N > 2/\bar{\delta}$ for a given interaction strength $\bar{\delta}$.

To draw an equivalence between an N-band and an N-chain system, let us first look for the generic dispersion relation for a system of N LLs arranged in the y direction and coupled with hopping strength t_y. The PBC in the y direction defines $p_y = \frac{2\pi}{N} i$, $i = 0, \pm 1, \ldots, N_{\max}$, where $N_{\max}$ takes both $\pm\frac{N-1}{2}$ values for odd N and only $+\frac{N}{2}$ for even N. Hence p_y can be used as a discrete index to rewrite the dispersion as

$$\epsilon_i(p) = t_i f(p) + c_i \tag{40}$$

where $p \equiv p_x$ and c_i is a function of $\frac{2\pi}{N} i$.

When N Schulz–Shastry chains are coupled by hopping, the generic dispersion is given by $\epsilon_i(p) = t_i p^2 + c_i$. Depending on the total number of particles, or equivalently the chemical potential μ given by $\epsilon_i(p) = \mu$, there are at most $2N$ Fermi points. This is similar to the case of N-band models where the dispersion is given by $\epsilon(p) = p^{2N} + \alpha_1 p^{2(N-1)} + \cdots$. By a proper choice of the $\{\alpha_i\}$, one can reproduce the same Fermi points and velocities (slope of $\epsilon(p)$ at the Fermi point) as required by any particular N-chain model. We have plotted the dispersions of a three-chain and a three-band model in figure 2 with appropriate choices of the $\{\alpha_i\}$. Note that the left and right branches are interchanged in the band labelled 2, the reason for which is explained in our previous work [13].

Also note that in our N-band or N-chain model, the fermions in one chain interact with the fermions of the opposite pseudospin in all the other chains. This is similar to the model studied by Bartosch and Kopietz [18], which had long-range interactions in the perpendicular direction and different from what happens in a real 2D system, where a particle in one chain hops or interacts only with neighbouring chains and the hopping probability or interaction strength decreases as the distance between the chains increases. Hence, the maximum number of chains $N_{\max}$ for which our model is similar to a realistic N-chain model is for $N_{\max} = 3$ with PBC. In that case, it is realistic to think that particles of one chain interact with particles

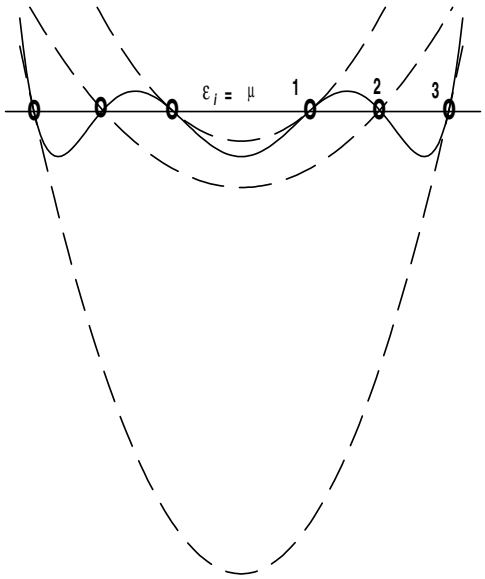

Figure 2. Three-chain system (broken curve) at a chemical potential μ is mapped onto a three-band model (full curve) in one dimension for low energies. Fermi points and the velocities are the same for both systems. Note that the slope at Fermi point 2 has to be compared with the left moving branch of the chain.

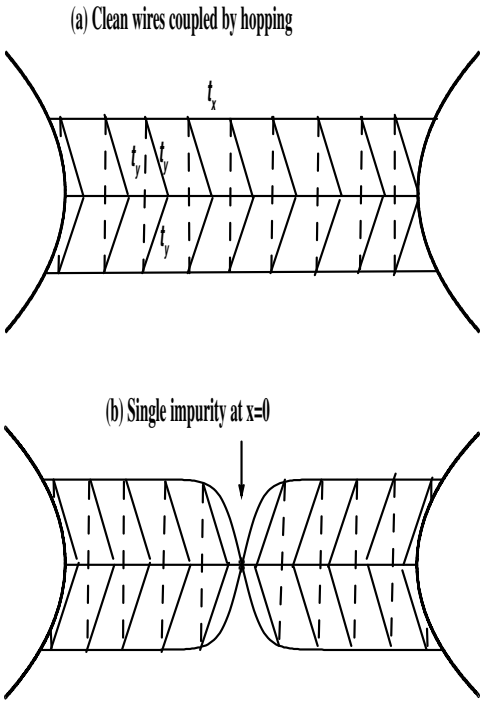

Figure 3. (*a*) Clean three-chain system with PBC, coupled to leads. (*b*) Crossed three-chain system with a single impurity (magnetic or non-magnetic) at the crossing point.

of all the other chains in the same manner, since all the chains are nearest neighbours of one another. This has been shown pictorially in figure 3(*a*).

So our analysis of an N-band model is fully relevant for a realistic three-chain model with PBC. Note that our model does not just allow hopping between chains (LLs coupled by hopping $\mathrm{LL}_{\mathrm{CH}}$) which could have been diagonalized, but it also includes interactions between the chains (interacting Luttinger chains LL_{I}). To drive the current in these systems we have to use probes as depicted in figure 3(*a*). The particle and the spin currents are J_{p} and J_{s} as we have defined earlier. At zero bias and in the large-L limit (without any impurities), the conductance is given by $G^N_{\mathrm{p/s}} = N\frac{2e^2}{h}(1 + N^2\bar{\delta}^2)$ as we have already computed in equation (35). For a system of $\mathrm{LL}_{\mathrm{CH}}$ the conductance would have been $G^N_{\mathrm{p/s}} = N\frac{2e^2}{h}(1 + \bar{\delta}^2)$. We have contrasted these conductances with the conductance of a system of N chains having free fermions (FL) and hopping, in figure 1, where the x-axis is N or equivalently the gate voltage V. (However, these conductances will be modified if the leads are FLs, just as for a single chain.)

The magnetic and non-magnetic impurities introduced in equations (14) and (29) for an N-band model can be thought of as the impurity potential at the crossing point of N-crossed LL wires. Ideally, in a multi-channel experimental system, an isolated impurity affects all the channels. In figure 3(*b*), we have shown the presence of an isolated impurity in a crossed three-chain system. The effect of this impurity on the three-chain system is exactly the same as that on a three-band model and we have computed this earlier. The non-magnetic impurity

remains marginal and hence the conductance can be calculated exactly. For both the particle and spin sector, we find $G^N_{\mathrm{p/s,imp}} = N\frac{2e^2}{h}(1+N^2\bar{\delta}^2)\frac{E_F}{E_F+\lambda^2}$ as computed in equation (38). For a magnetic impurity, it was shown that the strength flows to infinity for $\bar{\delta} < \bar{\delta}_c$, whereas for $\bar{\delta} < \bar{\delta}_c$ it depended on whether the barrier was weak or strong. Hence, conductance in the presence of magnetic impurities is given by

$$\begin{aligned} G^N_{\mathrm{p/s}} &= 0 && \text{for} \quad \bar{\delta} < \bar{\delta}_c \\ &= N\frac{2e^2}{h}(1+N^2\bar{\delta}^2) && \text{for} \quad \bar{\delta} > \bar{\delta}_c \quad \text{and} \quad \text{small } \lambda \\ &= 0 && \text{for} \quad \bar{\delta} > \bar{\delta}_c \quad \text{and} \quad \text{large } \lambda. \end{aligned} \tag{41}$$

If the isolated impurity or constriction affects different chains differently, then this analysis has to be suitably modified.

5. Discussion and conclusion

In this model, the strengths of the interactions of both the spin and the particle degrees of freedom are completely fixed by the gauge interaction. We have no freedom in changing the relevant strengths of the interactions, or in fact, explicitly changing the strength of the TR violation. However, it may also be of interest to study transport in systems which allow for arbitrary TR symmetry violation [19]. It should also be possible in such models to gradually change the strength of the TR violation and show that, in the limit where the TR violation goes to zero, the Kane–Fisher results are recovered.

A trivial generalization of the model is the following. For anisotropic interactions, i.e. when + particles of Ith chain interacts with − particles of Jth chain with strength $\bar{\delta}_{IJ}$, then the Hamiltonian will be the same that in equation (34) with $\tilde{\phi}_{\sigma I} = \phi_{\sigma I}$ and $\tilde{\theta}_{\sigma I} = \theta_{\sigma I} - \sigma\sum_J \bar{\delta}_{IJ}\phi_{-\sigma J}$. $\bar{\delta}_{IJ}$ has to be the same as $\bar{\delta}_{JI}$ from symmetry considerations. Thus, the whole analysis of section 3 may be repeated with

$$G^N_{\mathrm{p/s;clean}} = \frac{2e^2}{h}\left(N + \sum_{IJK}\bar{\delta}_{IJ}\bar{\delta}_{JK}\right). \tag{42}$$

It is easy to check that, when all the $\bar{\delta}_{IJ}$ are the same, we get back equation (35).

Another generalization worth pursuing [19] is to study resonant tunnelling through two impurities in these models. For non-magnetic impurities, one would expect the same results as for non-interacting fermions, since the impurity term is marginal. But for magnetic impurities, we would expect narrow resonances like for LLs, since a single impurity cuts the wire. More interestingly, when $N_+ - N_-$ on the island between the two impurities is odd (leading to non-zero pseudospin on the island), one may expect Kondo-type resonances. For the multi-chain models, here again it may be possible to have many more phases and perhaps non-trivial fixed points separating different phases.

In conclusion, we have studied transport in a class of exactly solvable models of interacting fermions, which are applicable not only to single-chain models, but also to multi-chain models up to $N = 3$, and have obtained the anomalous dimensions of several backscattering operators in the weak barrier and link limits. Since at low temperatures and low driving forces, the conductance has a power law behaviour in terms of the anomalous dimensions, we can directly obtain results for transmission and conductance. We emphasize that the study here is one of the first studies of transport through multi-chain LLs.

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